

SIMULATION TESTING OF TWO ESTIMATORS FOR THE ASSESSMENT OF SOUTHERN HEMISPHERE HUMPBACK WHALE BREEDING STOCK C AND ITS COMPONENT SUB-STOCKS

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ABSTRACT

This paper develops operating models of the C1 and C3 substocks of humpback whales in the western Indian Ocean which allow interchange between the two on the basis of the *Sabbatical* model for this mixing process. These operating models are used to compare the performance of the *Sabbatical* and *Resident* estimators, in what is intended as a preliminary exercise whose primary aim is to illustrate this simulation testing approach in the context of the substocks of breeding stock C of Southern Hemisphere humpback whales., .

KEYWORDS: HUMPBACK WHALES, SIMULATION TESTING, INTERCHANGE

INTRODUCTION

The Terms of Reference for the Intersessional Workshop on Assessment Methodology to take account of Mixing/Interchange between Southern Hemisphere Humpback Populations include to “review results from initial simulation testing of models put forward to estimate exchange rates and finalise further simulation tests to allow selection of appropriate models” (IWC, 2009).

This paper intends a contribution towards the first of these ends, through providing an illustration of the application of a simulation testing approach to models put forward to estimate exchange rates between the C1 and C3 breeding substocks of humpback whales in the western Indian Ocean. Butterworth and Johnston (2009) summarise four models put forward at a meeting held in Cape Town in December 2008 to represent the dynamics of these two populations and possible exchanges between them. Further, Johnston and Butterworth (2009) implement two of these models (*Resident* and *Sabbatical*) to estimate parameters for these populations, including the probability of interchange between them for the latter, using a Bayesian approach which takes account of capture-recapture information from photo-id data.

This approach in this paper follows that suggested at the December 2008 meeting reported in Butterworth and Johnston (2009). Operating models are developed based on the *Sabbatical* estimator, and used to test both the *Sabbatical* and *Resident* estimators. Four operating models are considered, crossing the factors of relatively low vs high exchange probabilities, and the current vs a considerably more intensive level of photo-id “captures”.

METHODS

The Operating Model

The Operating model (OM) is a *Sabbatical* model (i.e. allows for interchange on the breeding grounds – see Johnston and Butterworth (2009) for full model description). The following two sets of parameter values are used with the intention of being broadly similar to those that might apply in reality:

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	OM(1): $\alpha = 0.1$	OM(2): $\alpha = 0.3$
r^{C1}	0.10	0.10
r^{C3}	0.07	0.06
α^{C1}	0.1	0.3
α^{C3}	0.1	0.3
K^{C1}	6274	6001
K^{C3}	10197	8796

The K values were calculated using a backwards method to ensure that target abundance levels in 2006 for both stocks were 6000.

The above operating model produces N_{min} values for C1 and C3 of 459 and 625 respectively for OM(1) and 1711 and 597 for OM(2). These are well above the N_{min} constraints of 248 and 496 for C1 and C3 (the r^{C3} value was set lower than that for r^{C1} to ensure thus – to 0.07 for OM(1) and 0.06 for OM(2)).

The number of animals successfully photographed for the first time each year in each breeding area is either set equal to the numbers so photographed in reality, or to those numbers each multiplied by 5 (to examine how the precision of estimates of exchange probabilities in particular might be impacted by sample size).

In summary, four OM variants are considered:

OM(1): $\alpha = 0.1$; # of animals photographed for first time = #s photographed in reality,

OM(2): $\alpha = 0.3$, # of animals photographed for first time = #s photographed in reality,

OM(3): $\alpha = 0.1$; # of animals photographed for first time =5 times the #s photographed in reality, and

OM(4): $\alpha = 0.3$; # of animals photographed for first time =5 times the #s photographed in reality.

Each of these OMs was used to generate 100 pseudo-datasets for simulation testing purposes.

Data generation

For each simulation of the application of an estimation model, a pseudo-dataset is generated from the OM under consideration. This data set consists of the following elements, corresponding to the data used for the assessment conducted during the 2008 meeting of the Scientific Committee in Santiago IWC (2009):

1. 2003 Survey abundance estimate for C1 breeding grounds

$$\eta_{2003}^{C1,obs,sim} = \eta_{survey,2003}^{C1,true} e^{\varepsilon^{survey,sim}} \quad \text{where } \varepsilon^{survey,sim} \sim N(0,0.17^2)$$

$\eta_{survey,2003}^{C1,obs,sim}$ is the simulated data value for the C1 survey estimate of abundance in 2003 for simulation sim , and

$\eta_{survey,2003}^{C1,true}$ is the “true” value of the abundance of humpback whales on the C1 breeding grounds in 2003 obtained from the OM.

The CV of 0.17 assumed for the survey sampling variability is the estimate for the original survey (Johnston and Butterworth, 2009).

2. Cape Vidal SPUE for C1 breeding grounds

$$I_{SPUE,Vidal,y}^{C1,obs,sim} = I_{SPUE,Vidal,y}^{C1,true} e^{\varepsilon_y^{Vidal,sim}} \quad \text{where } \varepsilon_y^{Vidal,sim} \sim N(0,0.27^2)$$

$I_{SPUE,Vidal,y}^{C1,obs,sim}$ is the simulated data value for the Cape Vidal SPUE in year y for simulation sim , and

$I_{SPUE,Vidal,y}^{C1,true}$ is the “true” value for the Cape Vidal SPUE value in year y obtained from the OM by assuming equality to the abundance present in C1 at that time (as this is used as a relative index, specifying the constant of proportionality as 1 does not matter).

The years y here are the years in which these surveys viz. 1988, 1989, 1990, 1991 and 2002 actually took place. The CV of 0.27 for these SPUE indices corresponds to the standard deviation estimate (corrected for bias) of the residuals about a log-linear regression fit of the original estimates against year.

3. Aircraft SPUE for C1

The true expected number of whale sightings in year y is known from the OM (see Equation (13) of Johnston and Butterworth (2009)):

$$\hat{n}_y^S = q_{SPUE,aircraft} \eta_y^{C1,true} E_y$$

The probability of observing \bar{n}_y^S as follows:

$$p(\bar{n}_y^S) = \frac{(\hat{n}_y^S)^{\bar{n}_y^S} e^{-\hat{n}_y^S}}{(\bar{n}_y^S)!}$$

where $\bar{n}_y^S = 0, 1, 2, \dots, 20+$ (probability above 20 being negligible in practice and therefore lumped).

To generate the simulated data set of $\bar{n}_y^{S,sim}$ one first draws a random value Z from $U[0,1]$.

The cumulative probability for each n is $\bar{p}(n) = \sum_{k=0}^{k=n} p(k)$ is then calculated.

Finally, the realised $\bar{n}_y^{S,sim}$ is given by:

$$\text{IF } Z < \bar{p}(0) \quad \text{then} \quad \bar{n}_y^{S,sim} = 0$$

$$\text{IF } \bar{p}(k-1) \leq Z < \bar{p}(k) \quad \text{then} \quad \bar{n}_y^{S,sim} = k.$$

4. Capture-recapture data for C1 and C3

First, the probability of seeing an animal in a particular breeding ground and year is considered. These values are fixed across all simulations and are calculated as:

$$p_y^i = \frac{n_y^i}{\eta_y^{i,true}}$$

where

p_y^i is the probability of seeing an animal in area i in year y for (which is the same for all simulations),

n_y^i is the number of animals successfully photographed in region i in year y (which is the same for all simulations and equal to the number of sighted in reality), and

$\eta_y^{i,true}$ is the “true” number of animals in area i in year y in terms of the OM.

The $\hat{m}_{y,y'}^{i,j}$ values then follow from the OM and are the same for all simulations. The probability of observing $m_{y,y'}^{i,j}$ is then calculated as follows:

$$p(m_{y,y'}^{i,j}) = \frac{\hat{m}_{y,y'}^{i,j} m_{y,y'}^{i,j}}{m_{y,y'}^{i,j}!} e^{-\hat{m}_{y,y'}^{i,j}}$$

where $m = 0, 1, 2, \dots, 11+$ (probability above 11 being negligible in practice, and therefore lumped and truncated as 11).

To generate the simulated data set of $m_{y,y'}^{i,j,sim}$ one first draws a random value Z from $U[0,1]$.

The cumulative probability for each m is $\bar{p}(m) = \sum_{k=0}^m p(k)$ is then calculated.

Finally, the realised $m_{y,y'}^{i,j,sim}$ is given by:

$$\text{IF } Z < \bar{p}(0) \quad \text{then} \quad m_{y,y'}^{i,j,sim} = 0$$

$$\text{IF } \bar{p}(k-1) \leq Z < \bar{p}(k) \quad \text{then} \quad m_{y,y'}^{i,j,sim} = k.$$

The estimators

Two estimators are examined here – the *Sabbatical* and the *Resident* estimator. These estimators are described in full in Johnston and Butterworth (2009).

Simulation testing procedure

Each estimator is applied to the 100 generated datasets using the Bayesian methodology described in Johnston and Butterworth (2009). For each simulated dataset, the posterior median values of parameters of interest are stored. These are then finally summarised (across all 100 datasets) by calculating the medians of the 100 values for each such parameter. The results are reported in Tables 1a-d and compared to the OM “true” values. Tables 2a-d reports the RMSE (root mean square error) values of these posterior medians taken to provide the estimates of the quantities of interest.

RESULTS

Results for the *Sabbatical* and *Resident* model estimators when applied to data generated by each of the four OM variants are reported in Tables 1a-d. Tables 2a-d report comparisons of the RMSE (root mean square error) values for the two estimators. Figure 1 shows the distribution of the simulated data from OM(1) of the 2003 survey abundance data, whilst Figures 2a-d show the distributions of the simulated data for this same OM for the total numbers of recaptures within and between breeding area.

As the primary purpose of this paper is to illustrate the simulation testing approach in the context of the impact of C1-C3 interchange on assessment results, the results should not be seen as definitive, and accordingly only brief comment on their specific features is offered here. In broad terms there is no suggestion that the estimators are severely biased for any variable. For OM(1), the *Sabbatical* estimator tends to get r and current depletion (N_{2006}/K) too low, and K too high for the C1 population. If the *Resident* estimator (ignoring interchanges) is applied, there is a tendency towards values of r and current depletion for population C3 that are too low. For the higher interchange probability of OM(2), the bias in estimates of K and current depletion are much reduced, but the tendency to underestimate r remains. Increasing the number of captures (and hence recapture) does somewhat reduce the 90% range of estimates of interchange probabilities when this probability is 0.1 (OM(1) vs OM(3)), but such improvement is hardly evident for the larger interchange probability of 0.3 (OM(2) vs OM(4)). In terms

of RMSEs, the *Sabbatical* estimator tends to do better for r , current abundance and current depletion than the *Resident* estimator, though in the case of K for the C3 population the *Resident* estimator is generally the better.

REFERENCES

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Johnston, S.J. and Butterworth, D.S. 2009. Bayesian assessments of Southern Hemisphere humpback whale breeding sub-stocks C1 and C3. IWC document SC/F09/SH3.

Table 1a: *Sabbatical* and *Resident* model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to OM(1) generated data.

	“True” Values from OM(1)		<i>Sabbatical</i> Model estimator		<i>Resident</i> Model estimator	
	C1	C3	C1	C3	C1	C3
r	0.1	0.07	0.060 [0.020; 0.097]	0.057 [0.027; 0.083]	0.062 [0.026; 0.092]	0.044 [0.016; 0.064]
K	6274	10197	8088 [4588; 16849]	10129 [6508; 14548]	9874 [8453; 14407]	9992 [8790; 13302]
α	0.1	0.1	0.110 [0.010; 0.259]	0.138 [0.017; 0.293]	-	-
N_{min}	459	625	1061 [388; 3260]	901 [531; 2429]	841 [364; 2590]	947 [528; 2676]
N_{2006}	6146	6901	6144 [3831; 8442]	6996 [4483; 9538]	6728 [5313; 8110]	5327 [3924; 6890]
N_{2006}/K	0.980	0.677	0.810 [0.353; 0.997]	0.716 [0.425; 0.991]	0.675 [0.411; 0.901]	0.519 [0.323; 0.742]

Table 1b: *Sabbatical* and *Resident* model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to OM(2) generated data.

	“True” Values from OM(2)		<i>Sabbatical</i> Model estimator		<i>Resident</i> Model estimator	
	C1	C3	C1	C3	C1	C3
r	0.1	0.06	0.063 [0.018; 0.100]	0.067 [0.025; 0.087]	0.077 [0.021; 0.104]	0.043 [0.015; 0.058]
K	6001	8796	6380 [3056; 18130]	9819 [5676; 14686]	9234 [8142; 16117]	10029 [8695; 13200]
α	0.3	0.3	0.230 [0.026; 0.329]	0.322 [0.122; 0.390]	-	-
N_{min}	1711	597	1307 [455; 9657]	1121 [542; 2708]	1534 [687; 3618]	835 [518; 2603]
N_{2006}	6000	6000	4868 [1868; 9657]	7341 [3687; 11392]	8234 [6799; 9340]	4932 [3486; 6570]
N_{2006}/K	1.000	0.682	0.980 [0.274; 1.000]	0.791 [0.407; 1.000]	0.927 [0.473; 1.000]	0.486 [0.304; 0.706]

Table 1c: *Sabbatical* and *Resident* model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to OM(3) generated data.

	“True” Values from OM(3)		<i>Sabbatical</i> Model estimator		<i>Resident</i> Model estimator	
	C1	C3	C1	C3	C1	C3
r	0.1	0.07	0.066 [0.021; 0.099]	0.055 [0.025; 0.079]	0.066 [0.026; 0.096]	0.049 [0.019; 0.066]
K	6274	10197	7939 [4995; 16008]	10231 [722; 14217]	9646 [8334; 14466]	9788 [8720; 12954]
α	0.1	0.1	0.097 [0.010; 0.212]	0.111 [0.013; 0.217]	-	-
N_{min}	459	625	934 [386; 2876]	893 [529; 2437]	922 [391; 2759]	901 [522; 2705]
N_{2006}	6146	6901	6263 [4382; 8152]	6717 [4863; 8617]	7394 [6163; 8507]	5910 [4855; 6917]
N_{2006}/K	0.980	0.677	0.825 [0.384; 0.991]	0.672 [0.424; 0.933]	0.767 [0.466; 0.944]	0.596 [0.415; 0.756]

Table 1d: *Sabbatical* and *Resident* model estimator medians with 5th and 95th percentiles (i.e. results summarised across all 100 pseudo-datasets) when fitted to OM(4) generated data.

	“True” Values from OM(4)		<i>Sabbatical</i> Model estimator		<i>Resident</i> Model estimator	
	C1	C3	C1	C3	C1	C3
r	0.1	0.06	0.077 [0.016; 0.094]	0.067 [0.034; 0.078]	0.088 [0.027; 0.105]	0.043 [0.016; 0.057]
K	6001	8796	6729 [3265; 18822]	9819 [5676; 14548]	8697 [8138; 14559]	9997 [8899; 12793]
α	0.3	0.3	0.246 [0.011; 0.329]	0.33 [0.162; 0.390]	-	-
N_{min}	1711	597	991 [474; 3770]	731 [542; 2308]	1587 [784; 3494]	786 [516; 2373]
N_{2006}	6000	6000	4969 [1868; 9901]	6751 [3668; 10861]	8333 [7697; 9624]	4876 [3914; 5957]
N_{2006}/K	1.000	0.682	0.997 [0.233; 1.000]	0.780 [0.505; 0.998]	0.998 [0.592; 1.000]	0.484 [0.334; 0.624]

Table 2a: RMSE values of the *Sabbatical* and *Resident* estimators when fitted to OM(1) generated data.

	<i>Sabbatical Model estimator</i>		<i>Resident Model estimator</i>	
	C1	C3	C1	C3
r	0.042	0.014	0.042	0.025
K	2766	631	4184	467
α	0.041	0.066	-	-
N_{min}	774	385	677	382
N_{2006}	834	845	981	1743
N_{2006}/K	0.245	0.123	0.341	0.175

Table 2b: RMSE values of the *Sabbatical* and *Resident* estimators when fitted to OM(2) generated data.

	<i>Sabbatical Model estimator</i>		<i>Resident Model estimator</i>	
	C1	C3	C1	C3
r	0.038	0.010	0.035	0.058
K	1923	1670	4193	1269
α	0.096	0.038	-	-
N_{min}	641	843	568	330
N_{2006}	1589	2216	2149	1346
N_{2006}/K	0.223	0.193	0.231	0.211

Table 2c: RMSE values of the *Sabbatical* and *Resident* estimators when fitted to OM(3) generated data.

	<i>Sabbatical Model estimator</i>		<i>Resident Model estimator</i>	
	C1	C3	C1	C3
r	0.038	0.015	0.040	0.022
K	2738	610	4035	513
α	0.015	0.021	-	-
N_{min}	686	294	761	325
N_{2006}	477	286	1331	1007
N_{2006}/K	0.223	0.051	0.271	0.086

Table 2d: RMSE values of the *Sabbatical* and *Resident* estimators when fitted to OM(4) generated data.

	<i>Sabbatical Model estimator</i>		<i>Resident Model estimator</i>	
	C1	C3	C1	C3
r	0.038	0.007	0.026	0.017
K	2554	1199	3511	1217
α	0.082	0.032	-	-
N_{min}	689	485	514	231
N_{2006}	1572	1639	2384	1153
N_{2006}/K	0.251	0.125	0.135	0.199

Figure 1: Distribution of simulated data for OM(1) for the 2003 Survey abundance estimate for C1 breeding grounds ($\eta_{survey,2003}^{C1,true}$).

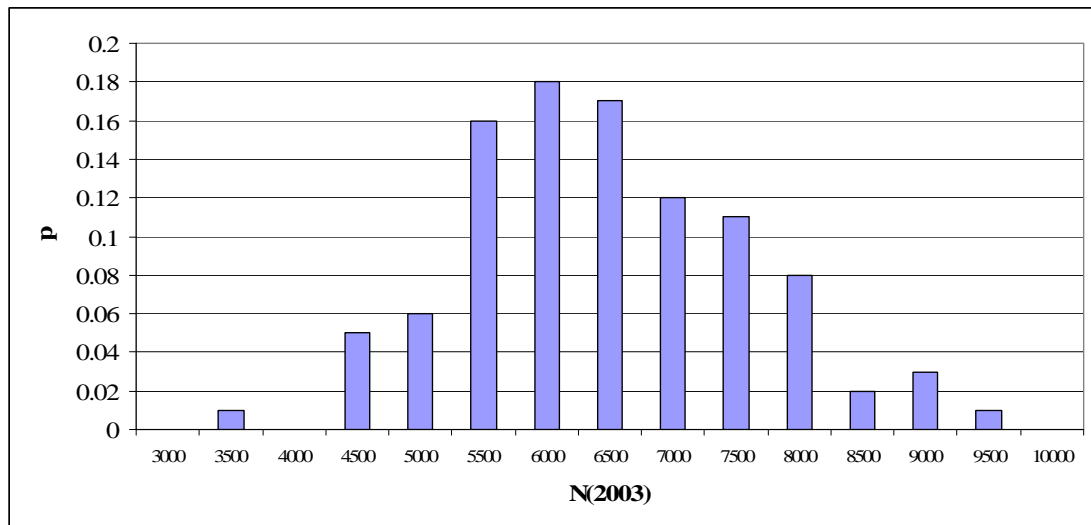


Figure 2a: Distribution of simulated data for OM(1) of the number of recaptures generated in C1 which were first seen in C1.

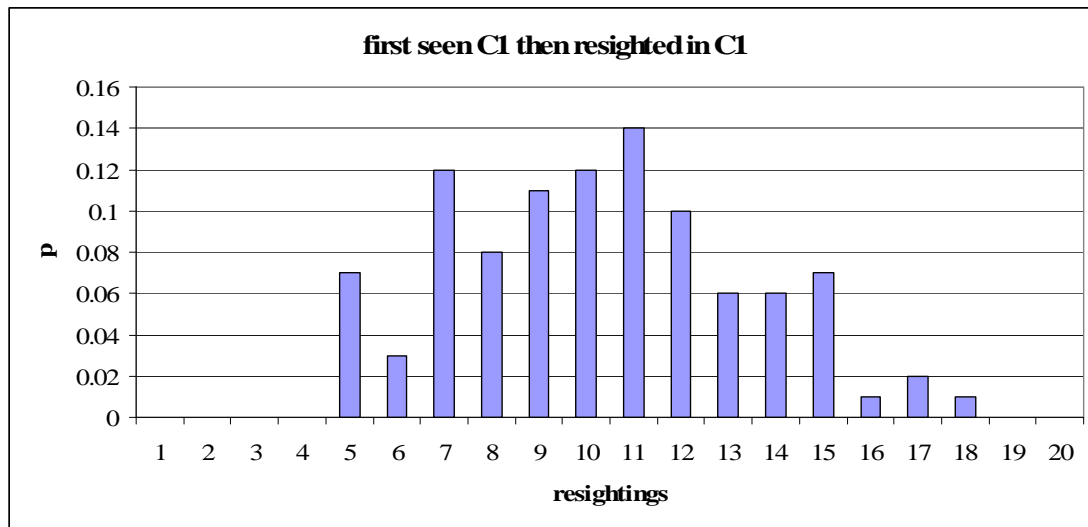


Figure 2b: Distribution of the number of recaptures generated in C3 which were first seen in C3.

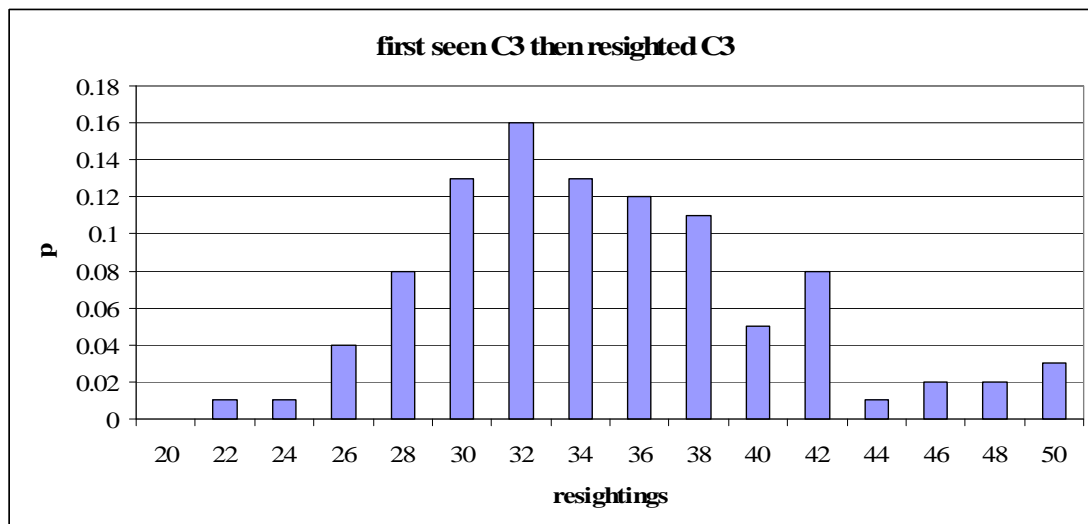


Figure 2c: Distribution of the number of recaptures generated in C3 which were first seen in C1.

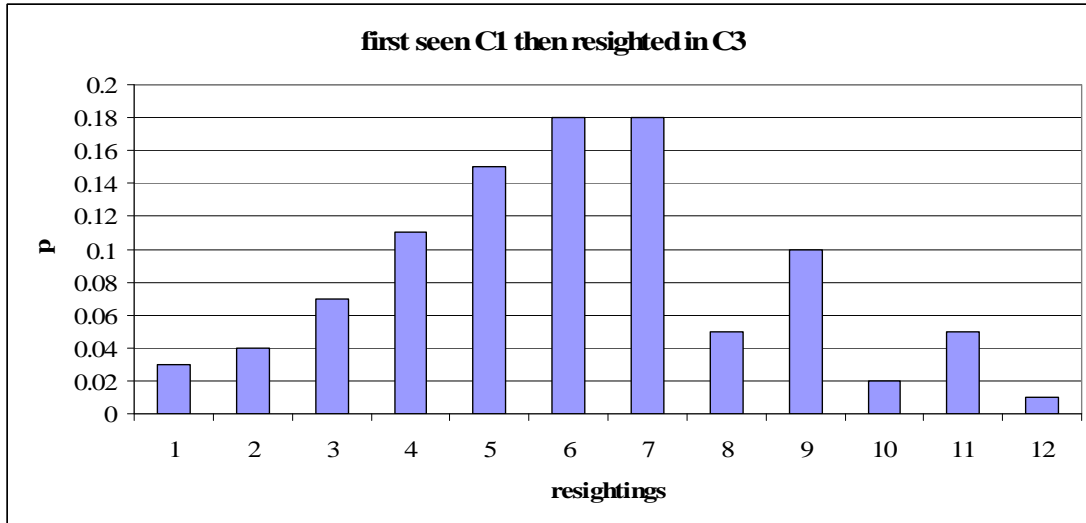


Figure 2d: Distribution of the number of recaptures generated in C1 which were first seen in C3.

